



Actually the Science of Music



FREQUENCY (PITCH)

Remember? The human ear has a frequency range from 20 Hz to 20,000 Hz
We measure frequency in Hz (Hertz) where Hertz is cycles per second.

Let's talk about frequency resolution; that is, what is the smallest frequency change we can detect at a given frequency?

Hearing tests have shown that at 1000 Hz we can detect a change of about 4 Hz
That's 4 in 1000 or 0.4 in 100 or 0.4 percent.

Our evolution figured out that we need this resolution across the frequency range.

At the low end (20 Hz) this gives a resolution of 0.08 Hz

At the high end (20,000 Hz) this gives a resolution of 80 Hz

That resolution range is 1000 times lower at the high end than at the low end.

We don't need 0.08Hz resolution at the high end. **Our head would have to be 1 metre wide to fit our ears in!**

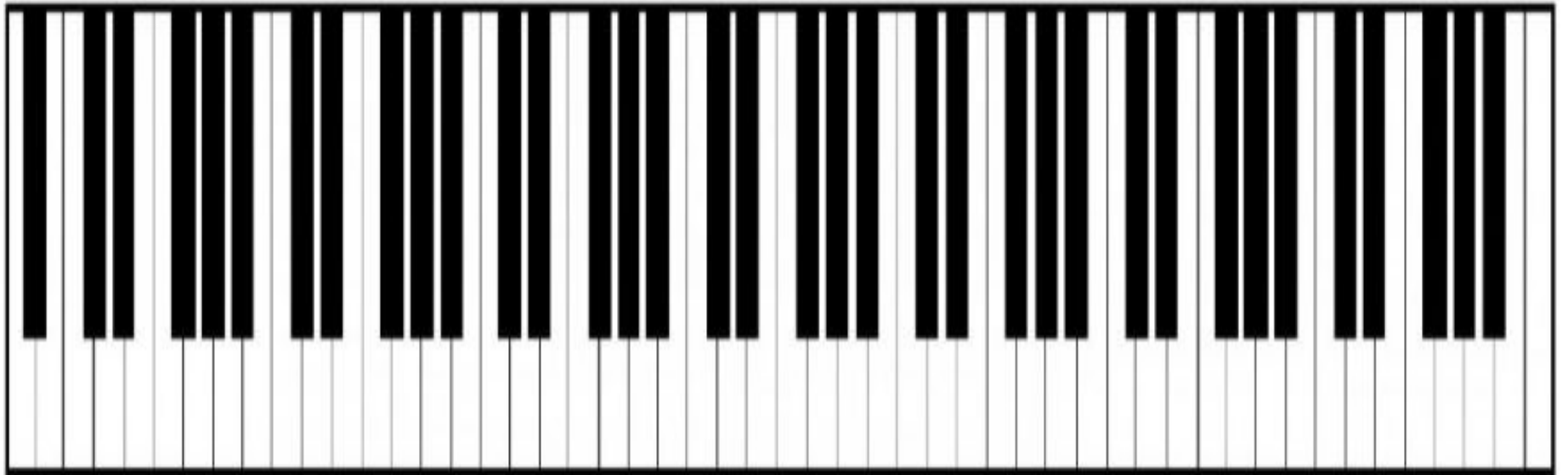
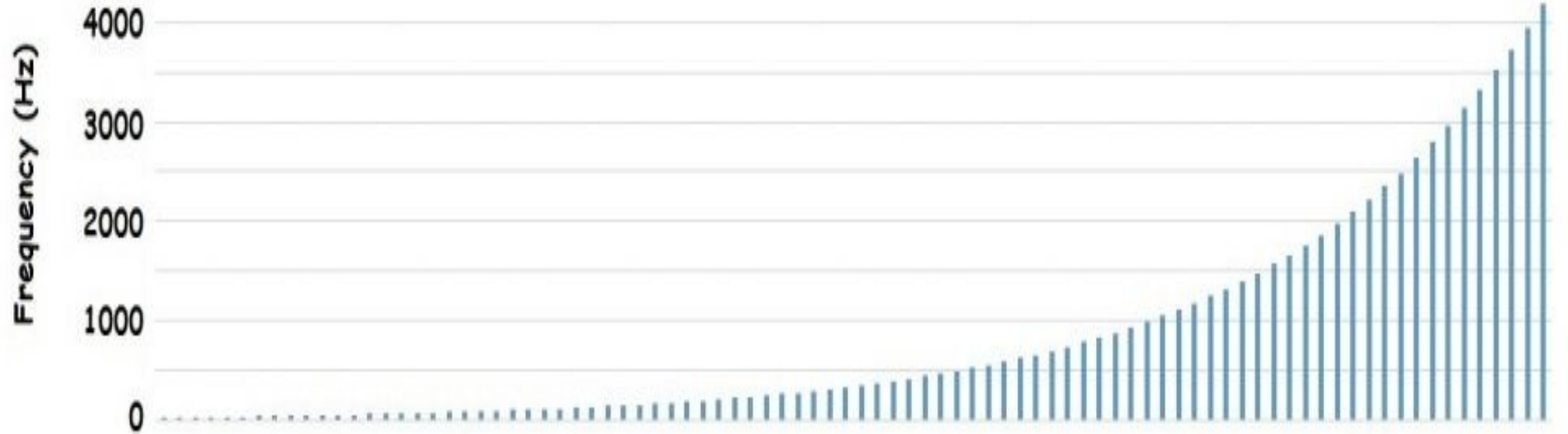
Thanks to Joseph Fourier, we know that a sound wave of a particular shape and frequency is composed of a sine wave of that frequency plus other sine waves of integer (2, 3, 4 etc) multiples of that frequency.

So evolution realised that hearing should incorporate a frequency resolution of a constant percentage and a nice way of incorporating integer multiples of frequency...

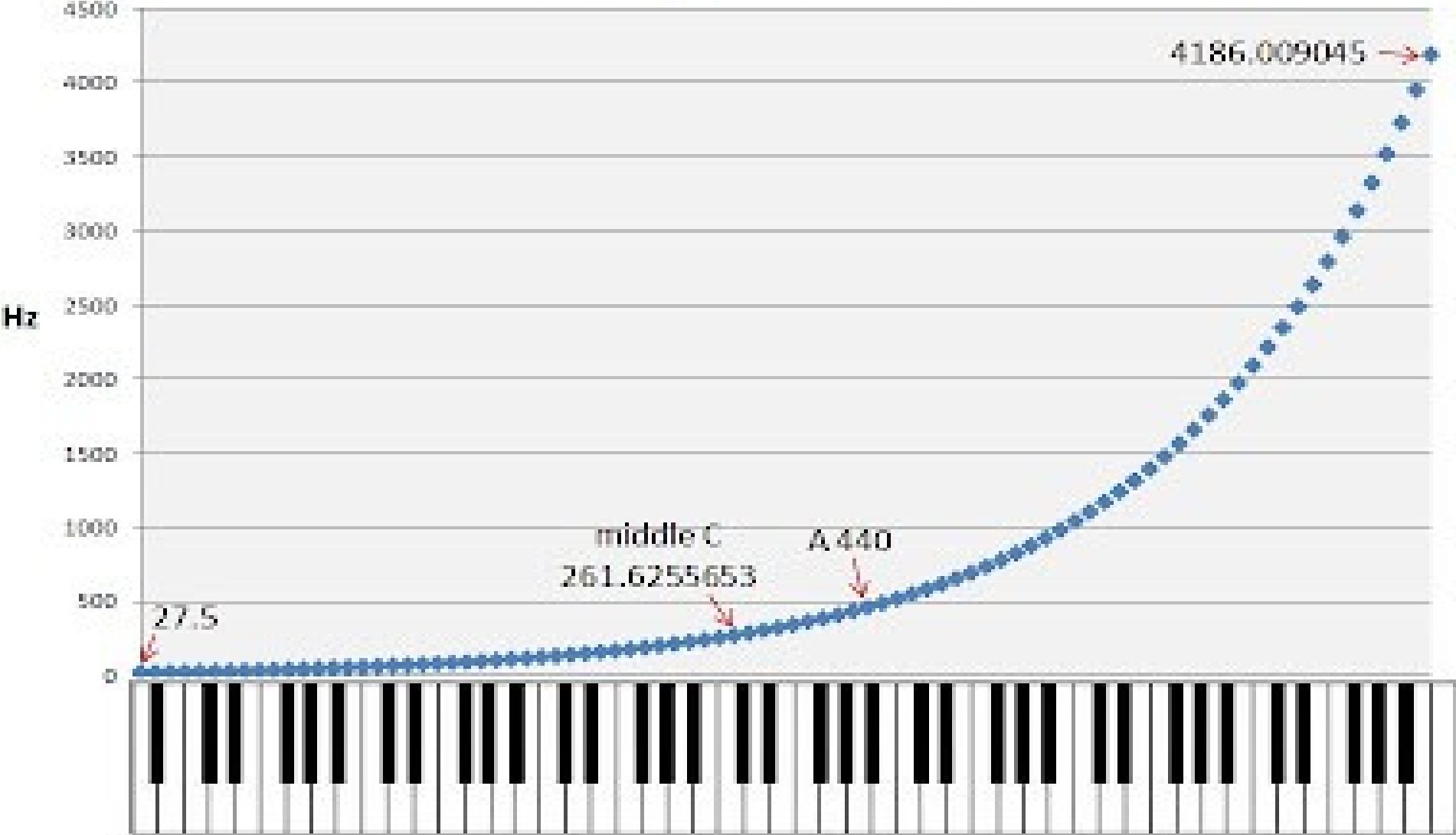
We have to thank the design for why we find music so enjoyable. Our ears are **“non linear”**.

When we hear a frequency that doubles then doubles then doubles... we hear it like the same musical note but higher and higher and higher. To a musician, this doubling is going up in octaves.

Notes on a piano versus their frequency



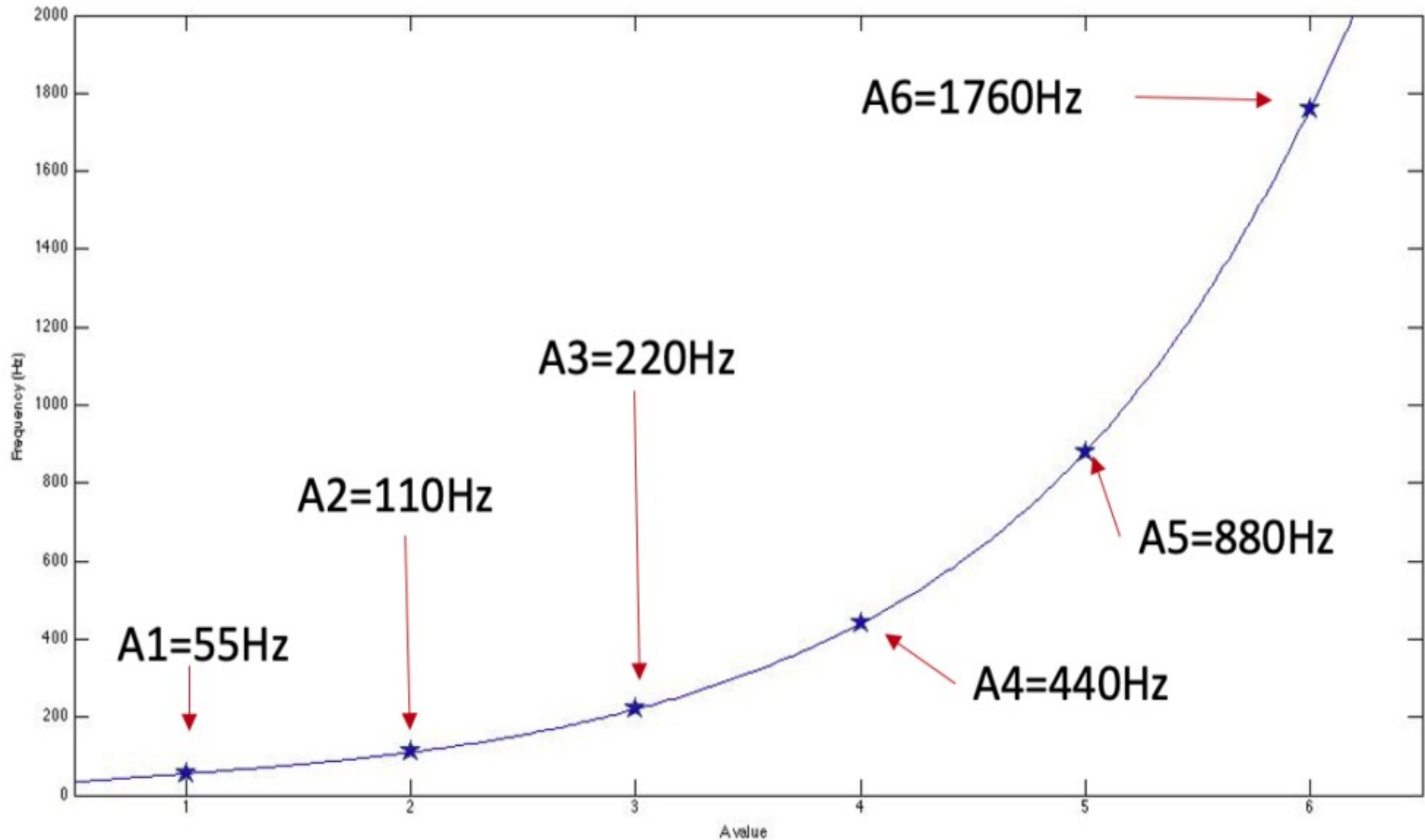
88 frequencies of the piano



The frequencies of musical note A in successive octaves.

The lowest octave A0 is not shown and has a frequency of 27.5 Hz

A4 at 440Hz is the international standard for tuning musical instruments



Middle C: 261.6 Hz



A: 220 Hz

A: 440 Hz

A: 880 Hz

AMPLITUDE (VOLUME)

Our ears can hear a pin drop but cope with the sound of a jet engine.

The range of sound power that our ears can deal with is 1 to 1,000,000,000,000

Guess what evolution did? It made our perception of volume “non linear”.

A perceived doubling of volume needs ten times the power.

You now have an excuse for asking for a 100 Watt audio amplifier for your bedroom – it’s only twice as loud as a 10 Watt amplifier!

We measure sound level in decibels. “deci” means one tenth. “bel” is named after the famous scientist Alexander Graham Bell.

Decibels (Db) are a non linear way of measuring and very useful for sound.

The smallest sound that we can hear is 0 Db. The loudest sound that we can bear is 120 Db.

A sound that is 30 decibels has ten times the power of a sound that is 20 decibels but only sounds twice as loud.

A sound that is 40 decibels has one hundred times the power of a sound that is 20 decibels but only sounds four times as loud.

Frequency (pitch) and Amplitude (volume) and their non linearities

These forms of non linearity are called **exponential**.

You might have heard of exponential growth in relation to the Covid virus or bird flu?

Exponential means that to get from one value to the next involves multiplication, rather than addition.

For example: multiplying 2 by 2, repeatedly gives numbers:

2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8182, 16384, 32768, 65536

The numbers get big very quickly.

Frequency Again

On a musical instrument, when going from one note to the next note higher the frequency actually goes up by a multiplying factor – in recent years always the same number but it was not always so...

Because our ears are non linear, from a musician's point of view we add notes as we go from one note to the next note higher.

Mathematicians call this change from multiplying to adding a “transformation” – clever stuff!

Our frequency range of 20 Hz to 20,000 Hz has 10 octave ranges as shown below (in Hz):

1	20 to 40
2	40 to 80
3	80 to 160
4	160 to 320
5	320 to 640
6	640 to 1280
7	1280 to 2560
8	2560 to 5120
9	5120 to 10240
10	10240 to 20480

So someone whose hearing “only” goes up to about 10,000 Hz has not lost half of the hearing range but just the top octave, that is, octave number 10.

Music

Musicians use letters (A, B, C, D, E, F, G), rather than numbers for notes, with the same letters repeated for each octave because going up from one octave to the next has the same sound (but higher!).

Actually there are 12 notes in an octave:

1	2	3	4	5	6	7	8	9	10	11	12
A,	Bb,	B,	C,	Db,	D,	Eb,	E,	F,	Gb,	G,	Ab

“b” means flat one note less than the letter above.

Sometimes “#” sharp (one note up) replaces “b”, example C# instead of Db. Why do we use this system?

C, D, E, F, G, A, B, C can also be called do, re, mi, fa, so, la, ti, do. That’s the C major scale. You don’t need any “b”s or “#”s. You can play this scale on a piano using only the white notes. Do you know the tune that starts “doe a deer, a female deer”?

Why start at C, not A?

A, B, C, D, E, F, G, A is the A minor scale. You don’t need any “b”s or “#”s. You can play this scale on a piano using only the white notes. A minor scale sounds more solemn than a major scale and was used by monks thousands of years ago for their singing worship.

Musicians talk about musical intervals. Musicians count all the notes when describing an interval so the interval C to G (C, D, E, F, G) is called a Perfect Fifth.

An interval starts with the first note of a scale. This note is called the “tonic” or “root”. Notes played together are called chords.

Science

Our ears like sets of frequencies that are low number fractions apart.

Two notes in a chord that are a Perfect Fifth apart have the higher note one and a half ($3/2$), that is, 1.5 times the frequency of the lower note.

Going up a further Perfect Fifth would be $3/2 \times 3/2$ times the original note; that's $9/4$.

Going up a further Perfect Fifth would be $3/2 \times 3/2 \times 3/2$ times the original note; that's $27/8$

Going up a further Perfect Fifth would be $3/2 \times 3/2 \times 3/2 \times 3/2$ times the original note; that's $81/16$

Hopefully you can see that these multiples are getting very wild and whacky! We never get back to a nice alignment within a musical octave.

Tuning used give nice sounding chords within each octave but chords spanning adjacent octaves (octaves next to each other) did not sound so good!

Keyboard music of the 15th century and earlier stuck to a range of notes within one octave as being brave enough to span multiple octaves gave rise to unpleasant sounding chords with so-called “**wolf notes**”.

This problem took until the year 1585 to resolve with **Equal Temperament Tuning**.

Equal Temperament Tuning

Science to the rescue!

The octave is still split into 12 notes but they go up in frequency by the same multiple from one note to the next.

The number is **1.05946309436**

Take 12 lots of the above number and multiply them together and you get the number 2.
So taking a number and multiplying it by the above number 12 times will give double the original number.

Take the frequency of the lowest note on a keyboard (A0). Repeated multiplications by the above number and saving each value give all the other notes.

The musical interval of a Fifth that should be $3/2 = 1.5$ becomes 1.498307

The frequency resolution of the human ear can't hear the difference, so we have a tuning system where you can go up and down in Fifths and other intervals and not get out of step with the notes on a piano or frets on a guitar etc.

This tuning took a long time to establish as many musicians thought it was “impure” to the ear and just not right!

Why are there 12 notes in an octave?

We have ten digits on our hands (two hands with four fingers and a thumb on each hand). So a number around ten would allow us a lot of freedom to make a range of notes (frequencies) within an octave.

We know that our ears like chords with notes that are nice fractions of one another.

Mathematically you get more nice fractions with numbers that have lots of “factors”. Factors are numbers that divide into another number by a whole number value.

Let’s look at factors of numbers around the number 10. Ignore the number one and the number itself:

Number	Factors
9	3
10	5, 2
11	None (prime number)
12	6, 4, 3, 2
13	None (prime number)
14	7, 2
15	5, 3

Look at the number 12. 12 wins! Twice the number of factors as the next best.

Terminology (Names) Used in Science and the Equivalent in Music

Science name

Frequency

Amplitude

Waveform (shape of wave)

Music name

Pitch

Volume

Timbre

Interval Ratios Compared – C Major Scale

		Tuning System			
	Interval Name	“Just”	“Equal Temperament”	Error (percent)	
C	Unison	1	1	0	
D	Major Second	$9/8 = 1.125$	1.122462	-0.21	(flat)
E	Major Third	$5/4 = 1.25$	1.259921	+0.79	(sharp)
F	Perfect Fourth	$4/3 = 1.3333$	1.33484	+0.12	(sharp)
G	Perfect Fifth	$3/2 = 1.5$	1.498307	-0.11	(flat)
A	Major Sixth	$5/3 = 1.6666$	1.681793	+0.91	(sharp)
B	Major Seventh	$15/8 = 1.875$	1.887749	+0.68	(sharp)
C	Octave	2	2	0	

Some errors are bigger than the 0.4 percent, as measured in hearing tests.

Hearing tests involve listening to alternating tones at very similar frequencies and judging whether they are different.

Music isn't like this so the above errors do not cause us to think that the tuning is awful!