Clever Carl

Carl was in year 4 at school. It was the start of a maths lesson.

The teacher asked the class to add up all the whole numbers (integers) between one (1) and one hundred (100). That is: 1+2+3+4+ ….and the rest!…. +97+98+99+100.

The teacher said that the first pupil with the correct answer would receive a gold star, to be presented at the next good work assembly.

The teacher was not expecting anyone to get the correct answer during the one hour lesson.

After less than one minute Carl put his hand up. Carl had the correct answer.

How do you think Carl did it?
Did Carl have a mind like a computer – no!

Clever Carl – The Method

1+2+3+4+ ….and the rest!…. +97+98+99+100

First – don't panic!
Try to break the problem into manageable parts.

The numbers increase by one (1) going up from the start but decrease by one (1) going down from the end. Can we pair up numbers from the beginning and end; adding the pairs to give the same value?

1+100 then 2+99 then 3+98 then 4+97 etc. Each pair add to give 101.

So we are going up from the start and down from the end, meeting the number range in the middle, is that ok?

How many numbers from 1 to 100?
Remember the gate post problem? There are 100 – 1 + 1=100.
100 is an even number so the numbers pair up. If there were an odd number then we would have one (1) left over, giving a problem that would need solving.

The number of pairs is 100/2=50, that is, half of 100.
The last pair, meeting in the middle of the number range is 50+51 still giving 101.

There will be 50 lots of 101. Numbers 1 to 50 pairing up with numbers 100 to 51.
We can check again: 1 to 50 has 50-1+1=50 numbers. 100 to 51 has 100-51+1=50 numbers.

This is looking good!

So we have 50 lots of 101. That's 101x50.
50 is 10x5, giving 101x10x5. To multiply by 10 in this case you can add a zero (0).

101x10=1010.

It just remains to multiply by 5. 1010x5=5050

To summarise, the answer is 101x50=5050

So 5050 is the answer

Clever Carl 2 – The General Method

1+2+3+4+ ….and the rest!…. +97+98+99+100

The original method of taking number pairs from each end only works for an even number range – otherwise we end up with one left over. Is there a method that works for ANY range of numbers? Yes!

How about the numbers from 3 to 101?

Let's do the range of numbers twice but in opposite order – like this:

3+ 4+ 5+ 6+ 7 ….and the rest!…. 97+98+99+100+101
101+100+99+98+97 ….and the rest!…. 7+ 6+ 5+ 4+ 3

3+101 is the same as 4+100 and 5+99 and 6+98 and 7+97 and for the rest.

**** There must be an overall divide by 2 in the formula because we are doubling the total number. ****

We can see that we have in the example:

(101+3) x (how many numbers there are in the range 3 to 101) / 2

In a general case this is:

(biggest number + smallest number) x (how many there are in the range) / 2

How many are there in the range? Remember the gate post problem!
It's (biggest number – smallest number + 1)

So the general formula is:

(biggest number + smallest number) x (biggest number – smallest number + 1) / 2

Let's call the biggest number "B" and the smallest number "S" to save space!

(B+S) x (B-S+1) / 2

That's Algebra! Algebra where you know what B and S are – not one of those Algebra problems where you have to work out what B and S are.

Who Is Carl?

Carl Friedrich Gauss 1777 to 1855
https://en.wikipedia.org/wiki/Carl_Friedrich_Gauss